

Review¹ of
Famous Puzzles of Great Mathematicians
by Miodrag S. Petković
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1 Introduction

Famous Puzzles of Great Mathematicians contains a nice collection of recreational mathematics problems and puzzles, problems whose solutions do not rely on knowledge of advanced mathematics. These problems mostly originated from great mathematicians, or had at least captured their interest, and hence the title of the book. Despite its recreational nature, this book does not give up on being rigorous in its arguments, nor does it shy away from presenting some difficult problems, albeit solvable by elementary methods. And even though the math in this book will not be new to most readers of this column, this is not the book's purpose. Among other reasons Petković states the book was written to attempt “to bring the reader closer to the distinguished mathematicians” and to “show that famous mathematicians have all communicated brilliant ideas ... by using recreational mathematics.” My review is written with this in mind.

2 Summary

After giving an overview of recreational mathematics, Petković divides the majority of the book's puzzles into 9 topics, each getting its own chapter, each of which I summarize herein. Each chapter not only contains a collection of problems, but is also replete with quips, anecdotes, and short biographies. Petković's book follows the pattern of first giving an introduction to and history of an area, then giving a couple challenge problems whose solutions he works out, and finally leaving the reader with some challenge problems, the solutions to which appear at the end of their respective chapters.

Following the topics chapters is an extra chapter that has additional challenge problems from various mathematicians – the answers to these problems are not given in the book. Then, there is an appendix elaborating on certain topics appearing in the book. The book concludes with short biographies of the mathematicians mentioned throughout.

2.1 Arithmetics

This chapter contains many puzzles focused on numerically computing a given result – from familiar problems involving finding a person's age, to problems involving Fibonacci numbers and other series, to problems asking the reader to do “reverse computations” such as finding the sides of a shape given its area. Most of these problems require either seeing some trick and then setting up an equation

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or a recurrence to get the solution. These arithmetic problems are some of the most accessible to a layperson. In this chapter, Isaac Newton's biography receives the most (well-deserved) space. He is credited with solving puzzles of the form "If x people can do y tasks in z hours, how many people would it take to do ...?" Some challenge problems of this form are presented at the end of the chapter.

2.2 Number Theory

This chapter covers a large variety of problems from the work of many important mathematicians, from Archimedes to Euler, Dirac, and Ramanujan. This is unsurprising as number theory is known for producing many interesting problems that are both immediately enticing and easy to understand. The number theory problems in this chapter also give an opportunity to demonstrate the usefulness of computers for solving mathematical problems. Petković explains how the solution to a seemingly simple problem posed by Archimedes counting a farmer's cattle wasn't solved until 1965 by a clever use of IBM digital computers. Problems in this chapter all have a similar feel – of finding numbers that satisfy certain properties, and solutions to them often call for heavy use of computation.

2.3 Geometry

The geometry chapter is the longest in this book and contains questions that at first seem like they belong in a high school geometry class, but end up being harder and more interesting. Two compelling examples from this chapter are the problem of finding the best latitude for looking at Saturn's rings and the problem of determining the maximum number of parts a space can be divided by n planes. Of more relevance to computer scientists, Petković shows the solution the problem of connecting 4 villages on the corners of a square by the smallest road system – an instance of the Euclidian Steiner tree problem.

2.4 Tiling and Packing

This chapter, as its name suggests, focuses on problems in filling a plane or 3-dimensional space with objects. Here, Petković gives an extensive history of progress in tiling problems. One example is the progress in finding a non-periodic tiling of the plane – at first it was thought impossible to do, then a solution was given using 2000 shapes, and the number of shapes needed for an aperiodic tiling kept being reduced until Roger Penrose got the number down to only 2. John Conway and Donald Knuth also expectedly make an appearance. The tiling and packing problems in this chapter all have a similar feel, and the questions are mostly challenge questions.

2.5 Physics

In a somewhat nonstandard turn for recreational mathematics books, Petković includes a short chapter of problems arising from physics. His definition of physics is quite broad, and many problems inspired by real-world situations count, including the famous puzzle of calculating the distance a fast object travels while bouncing between a wall and a slower object moving towards the wall – included with this puzzle is the well-known story of von Neumann not being able to see the trick, yet solving it quickly nonetheless. This chapter also contains an interesting problem of a man being trapped inside a ring with a lion, both of whom can run at the same speed. The

question is whether the man can outrun the lion forever, and I won't spoil it by putting the answer here.

2.6 Combinatorics

The combinatorics chapter is full of various counting problems, so it is hard to do it justice with a summary. Here, Petkovič puts even the most familiar problems into context. For example, he points out that the problem of counting the number of ways of choosing k objects from a set of n was posed and calculated by Mahāvira, an Indian mathematician, in 850 A.D. Then, he moves on to a variety of topics: finding good arrangements, Gray-codes, Eulerian squares, Cayley's recurrence for counting trees, the towers of Hanoi, and many other problems. And he does not neglect to include a short and entertaining biography of Paul Erdős.

2.7 Probability

This short chapter contains some famous mathematical gems. As Petkovič points out, "probability is full of surprising results and paradoxes, more than any other branch of mathematics." He includes problems on how to divide stakes when a game is interrupted (considered by both Pascal and Fermat), the gambler's ruin problem, St. Petersburg paradox (a game with low winnings but infinite expectation), among others.

2.8 Graphs

Petkovič gives graph theory problems a chapter of their own. He begins by covering traditional problems, including the Königsberg bridge problem and Eulerian and Hamiltonian tours, making a nice connection between the latter and the towers of Hanoi, which are covered earlier in the book. He then moves onto less traditional graph problems, (but which are traditional recreational mathematics problems,) including river-crossing problems, in which a person has to get a wolf, goat, and cabbage across a river without anything being eaten and measurement problems, where you have to measure some quantity of water with seemingly ill-suited beakers for the task.

2.9 Chess

Chess problems at first seem out of place in this book, but Petkovič explains how they captured the attention of some famous mathematicians, from Euler to Knuth. These problems are versions of puzzles familiar to many chess players and recreational mathematicians and involve knights tours and attacking queens. Gauss's biography surprisingly makes an appearance in this, of all, chapters, but his apparent interest in the well-known eight queens problem explains the choice.

3 Opinion

This book contains a nice representative set of recreational problems spanning many different areas of math. What makes this book especially compelling is Petkovič's efforts in putting the problems into context. He makes it clear that math is a human subject, with its own stories and history. The biographies of the mathematicians involved are witty and perhaps the best part of the book – this is by no means a criticism of the main content but a testament to Petkovič's extensive research

and good storytelling. I also think that the problems selected are quite appropriate. This book is very easy to follow, and the problems are of varying difficulty.

It does not mean I do not think there is room for improvement. Some of the puzzles are stated in their original form (as posed centuries ago) and are hard to understand. This can be a positive for historical reasons, but also a bit confusing – I sometimes had to look at the answers and discussion to understand what a question is asking. Furthermore, how the problems were divided among the chapters seemed arbitrary at times.

But most disappointing to me is that Petković avoids discussing most core theoretical computer science ideas (which surely belong within mathematics), even when some problems beg for it. To give just a few examples – he presents the traveling salesperson and Hamiltonian path problems without mentioning NP-completeness, gives an example Euclidian Steiner tree problem without talking about its computational complexity, and has an entire chapter on tiling problems without mentioning undecidability. By the end of reading the book, a reader may be left with the impression that computers are only interesting insofar as they help search through many possibilities or can compute the solution to an otherwise intractable equation. And even though computers and algorithms are often mentioned, it is unfortunate that computer science theory is not properly included in this otherwise encyclopedic collection, and I am left with the feeling a good opportunity to introduce readers its mathematical beauty was lost.

Of course, this is not a computer science book, and it is unfair to judge this book solely from that lens, so I still wholeheartedly recommend it to a wide variety of audiences. While professional mathematicians (and theoretical computer scientists) would know the math behind the problems this book, I suspect most would discover a few new interesting puzzles and some history, especially outside their main fields. To this audience, this book could be a fun and easy read. It might even give some inspiration for problems to present in lecture, though I doubt that is the best use of this book.

Mainly, I would recommend this book to anyone interested in recreational math or anyone who likes math puzzles. Petković aims to bring his readers closer to the ideas of brilliant mathematicians, and I believe he succeeds. This book would be especially appropriate for undergraduates or even high school students with aptitude in mathematics. They should find *Famous Puzzles of Great Mathematicians* both very informative and fun, and might even become inspired to explore a career in math.